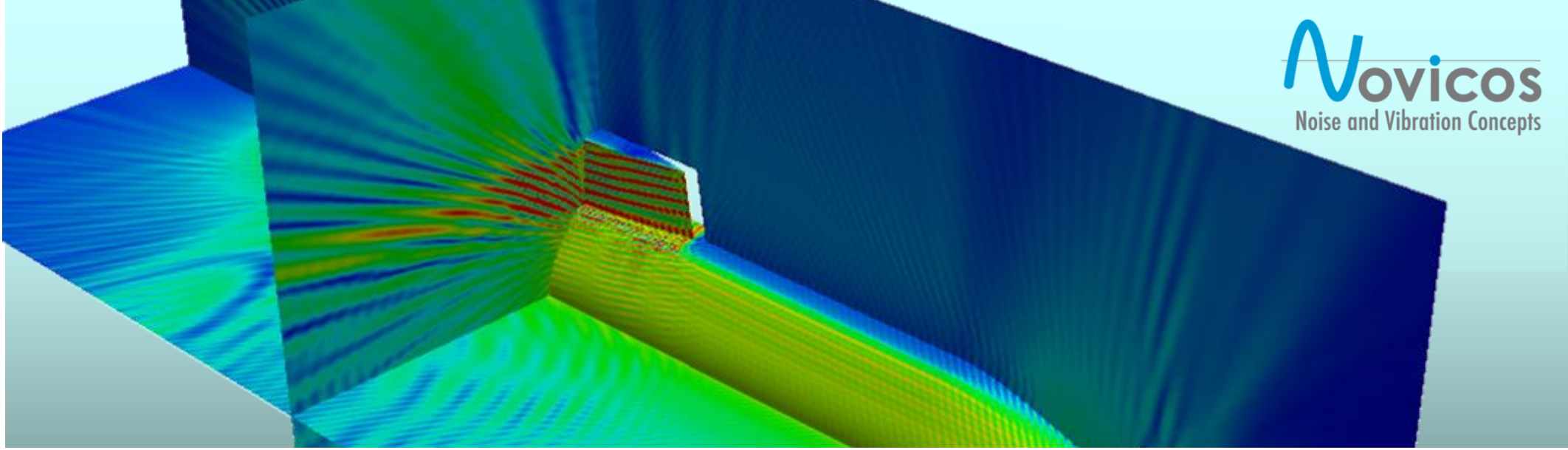


Novicos
Noise and Vibration Concepts

On the efficient use of the hierarchical matrix BEM for target echo strength simulations

B. Dilba, M. Markiewicz, O. von Estorff

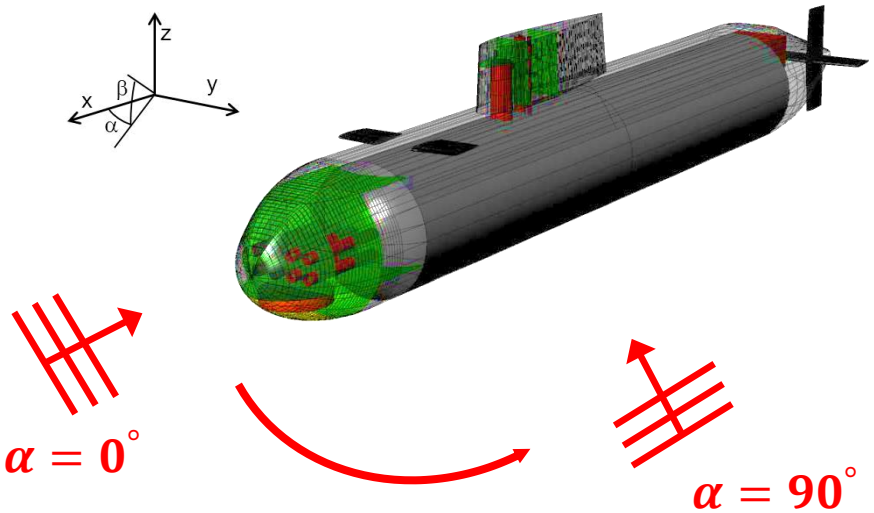
Content:

- Target Echo Strength
- BEM exterior problem
- Fast BEM – H-Matrix compression
- BeTSSi TES simulation
- Solution strategies
- Field point evaluation

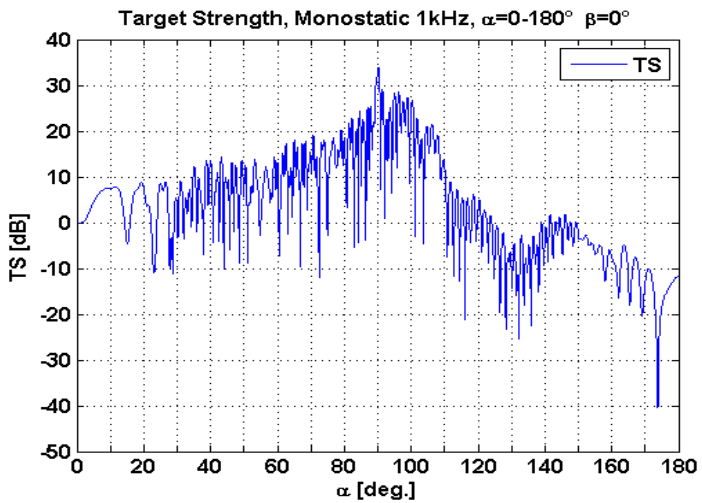
Target Echo Strength (TES)

- TES is a measure of an object‘s ability to return an echo
- Defined by the backscattered acoustic pressure due to an impinging wave
- Crucial design parameter for submarines
- Understanding of the TES properties is very important at early stages of development

  ➔ **numerical methods (today: BEM)**



Submarine model of **B**enchmark **T**arget **S**trength **Si**mulation (BeTSSi) workshop, organized by WTD71 (2014), DRDC, TNO (2016)



BEM - exterior acoustic problem

Governing equations

- Helmholtz equation with Neumann b.c. (q on $\partial\Omega$)

$$\Delta p(\mathbf{x}) + k^2 p = 0 \qquad \mathbf{x} \in \Omega$$

$$\frac{\partial p(\mathbf{x})}{\partial n(\mathbf{x})} = q(\mathbf{x}) \qquad \mathbf{x} \in \partial\Omega$$

$$\frac{\partial p}{\partial r} - ikp = 0 \qquad r \to \infty \text{ (Sommerfeld radiation condition)}$$

- Transformation to ***Kirchhoff-Helmholtz*** Boundary Integral equation (BIE):

$$c(x)p(x) + \int_{\partial\Omega} \frac{\partial G(x,y)}{\partial n(y)} p(y)\partial\Omega(y) = -\int_{\partial\Omega} G(x,y)\frac{\partial p(y)}{\partial n(y)}\partial\Omega(y) \qquad x \in \partial\Omega$$

- Discrete Boundary Integral equations (frequency dependent):

$$\boxed{\left(\frac{1}{2}I + K\right)p = Vq \qquad \textbf{(CBIE)}}$$

$$\boxed{\left(\frac{1}{2}I - K'\right)q = -Dp \qquad \textbf{(HBIE)}}$$

Solve: $p = \left(\frac{1}{2}I + K\right)^{-1} Vq$ or $p = -D^{-1}\left(\frac{1}{2}I - K'\right)q$

➡ **singular for some k (→irregular frequencies)**

BEM - exterior acoustic problem
Regularisation

- Direct Combined Field Integral Equation (DCFIE):

$$\left(\frac{1}{2}\mathrm{I} + \mathrm{K} - \boldsymbol{\alpha}\mathrm{D}\right)\mathrm{p} = \left(\mathrm{V} - \boldsymbol{\alpha}\left(\frac{1}{2}\mathrm{I} - \mathrm{K}'\right)\right)\mathrm{q}$$

choice of $\boldsymbol{\alpha}$:

– uniquely solvable for all real valued k if:

$\mathfrak{Im}(\boldsymbol{\alpha}) \neq 0$ ***(Burton and Miller (BM))***

– optimal with respect to the condition number:

$\boldsymbol{\alpha} = \frac{\mathrm{i}}{\mathrm{k}}$ ***(Kress)***

– ideal in terms of eigenvalue clustering:

$\boldsymbol{\alpha} = \boldsymbol{NtD}$ (**N**eumann-**t**o-**D**irichlet) from **Calderón Identity**

$$\boldsymbol{NtD} = \mathrm{D}^{-1}\left(\frac{1}{2}\mathrm{I} - \mathrm{K}'\right) \quad \rightarrow \quad -\boldsymbol{NtD}\mathrm{D} = \left(\frac{1}{2}\mathrm{I} - \mathrm{K}\right) \quad \text{with} \quad \mathrm{K}'\mathrm{D} = \mathrm{DK}$$

$$\Rightarrow \left(\frac{1}{2}\mathrm{I} + \mathrm{K} - \boldsymbol{NtD}\mathrm{D}\right) = \mathrm{I}$$

$$\Rightarrow \boldsymbol{\alpha} = \boldsymbol{NtD} \approx \frac{\mathbf{1}}{\mathbf{ik}}\left(\mathbf{1} + \frac{\Delta_\Gamma}{\mathbf{k}_\varepsilon^2}\right)^{-1/2}$$

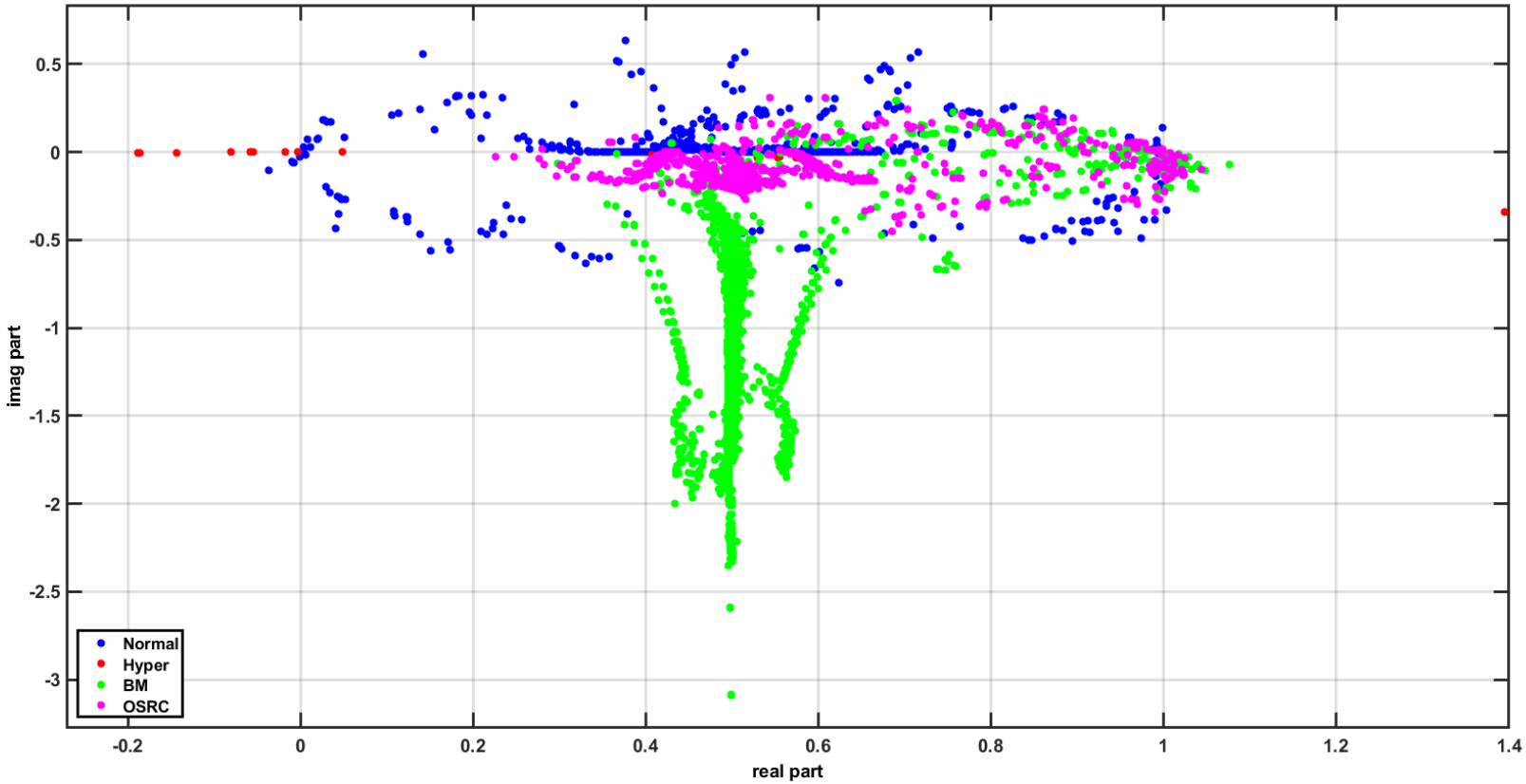
On Surface Radiation Condition (OSRC)

- Efficient preconditioner for iterative solvers e.g. GMRES (faster convergence)

– physical coordinates p and q are maintained.

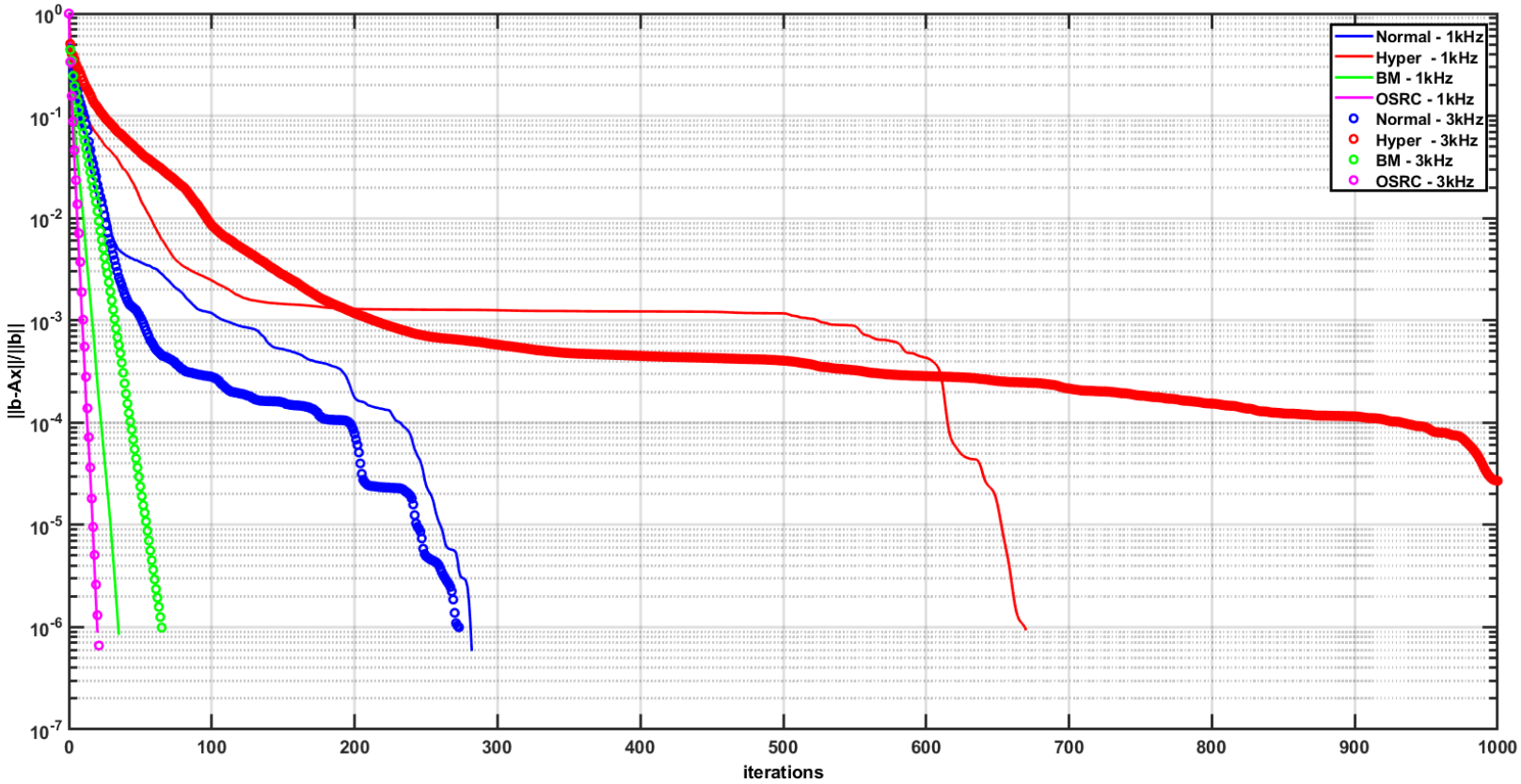
BEM - exterior acoustic problem

Effect of regularisation (Cat‘s eye)



BEM - exterior acoustic problem

Convergence of GMRES (Cat‘s eye)

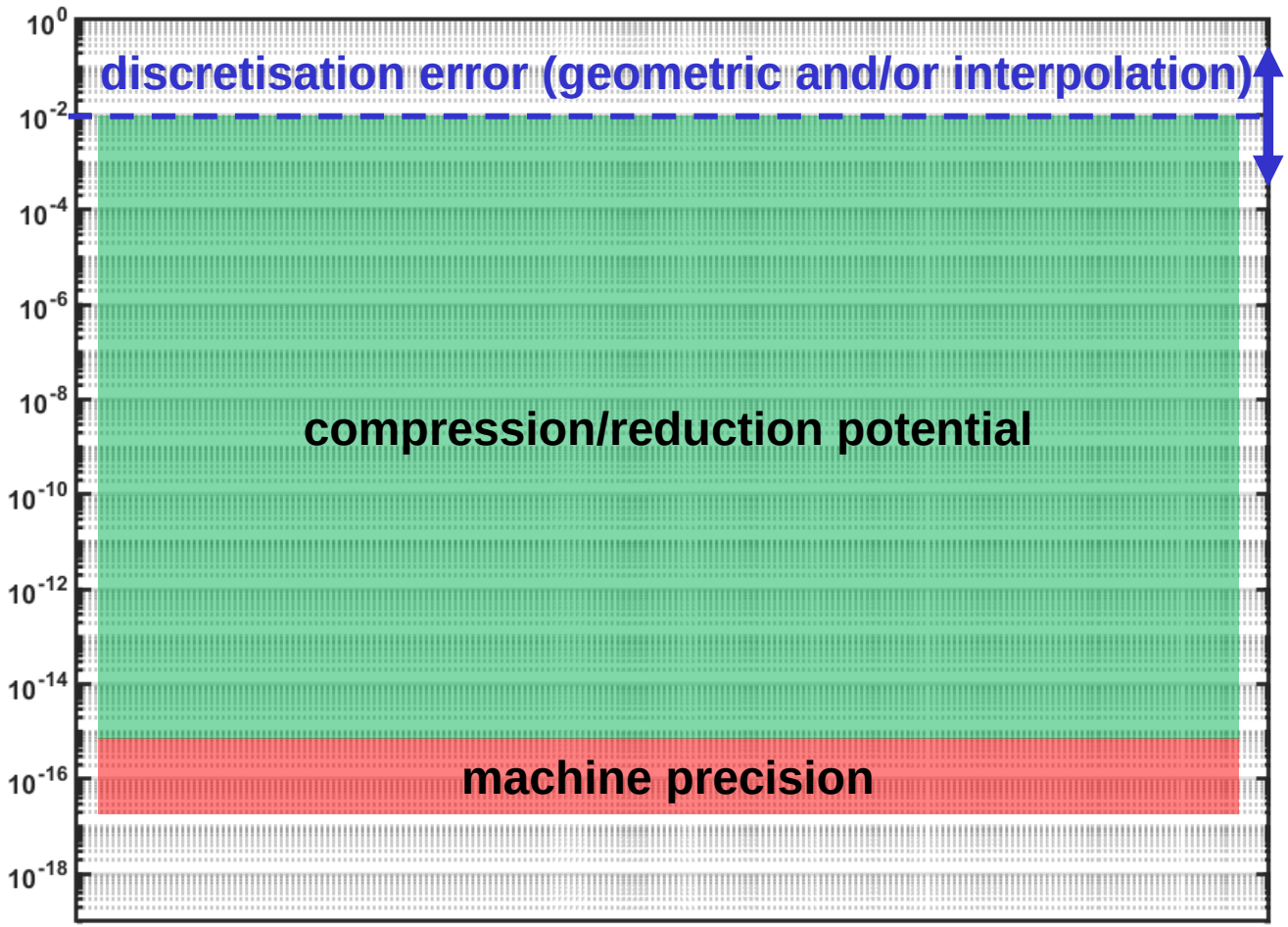


Fast BEM - H-Matrix compression
Motivation

- Exterior BEM problems for large scale structures lead to large scale problems.
  - $\mathrm{n} = 7.9 \times 10^6$ DOFs: Submarine (length 60m) at 10kHz
  - Standard BEM requires **908TB Memory** $\mathcal{O}(\mathrm{n}^2)$, … **solution time** $\mathcal{O}(\mathrm{mn}^2 - \mathrm{n}^3)$
- Matrix compression techniques reduce the overall complexity
  - FMM, H²- and H-Matrix lead to $\mathcal{O}(\mathrm{n}\log(\mathrm{n}))$ for arithmetic operations, e.g. MVP
    - Hardware acceleration with GPUs (setting up the system matrix)
    - Plane wave approach for high frequencies FMM and directional H²-Matrix (DH²)
    - Exploiting properties of the surface
    - …
- Efficient solution process becomes more and more important, especially for large scale problems with high MVP-costs
  - Direct solution with hierarchical LU or $\mathrm{LDL}^{\mathrm{T}}$ (only for many right hand sides)
  - Improving convergence behaviour of iterative solvers (e.g. GMRES)
    - Construction of preconditioners (algebraic, analytic)
- Improvements for TES-Simulation with the H-Matrix BEM approach

Fast BEM - H-Matrix compression

Basic idea – approximation of arithmetic operations (MVP,MM)

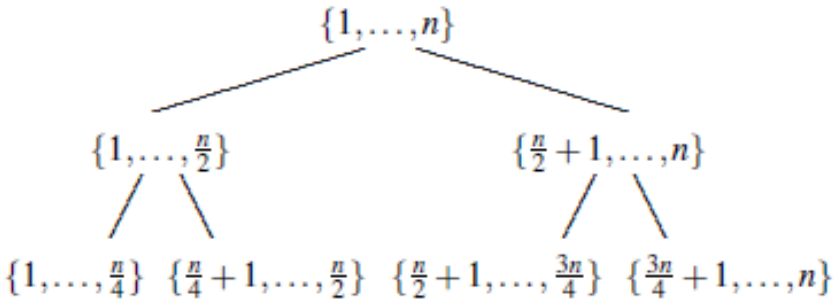


Introduced by BEM $\mathcal{O}(\mathbf{n}\,\log(\mathbf{n}))$

Matrix entries $\mathcal{O}(\mathbf{n}^2)$
Matrix-Vector-Product
Matrix-Matrix-Multiplication

Hierarchical Matrix

1. ***Index cluster Tree: T_I***
   Hierarchical clustering of matrix index set according to ***principal component analysis (pca)***



well-balanced index cluster tree T_I

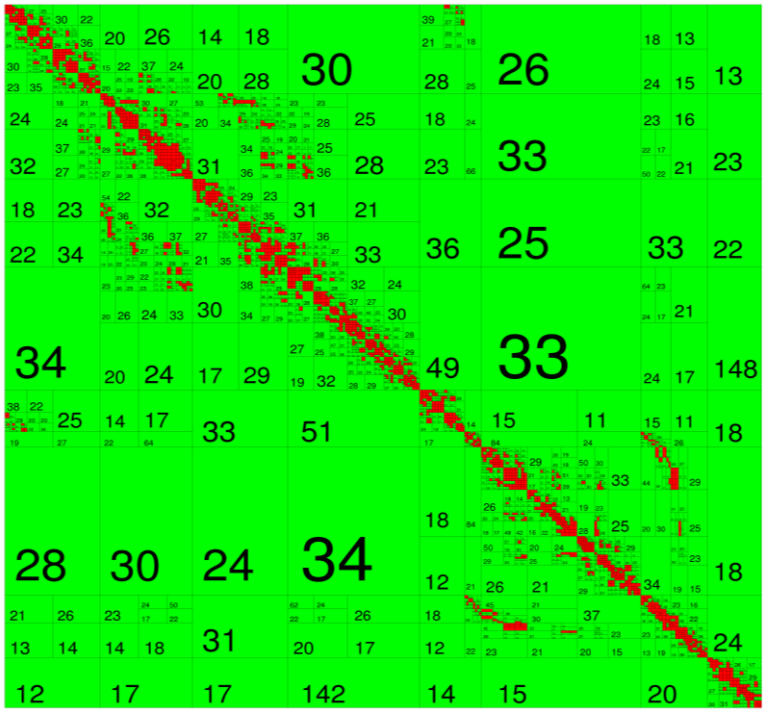
2. ***Block cluster tree: $T_{I \times I} = T_I \times T_I$***
   Matrix partition ***P*** generated by checking the admissibility condition for row and col clusters X_t and X_s:

$$\min\{ \operatorname{diam}(X_t), \operatorname{diam}(X_s) \} \leq \eta \operatorname{dist}(X_t, X_s)$$

3. ***Block approximation***
   If the admissibility condition is fulfilled the admissible matrix block can be approximated by a **low rank** matrix

$$A = \sum_{i=1}^{l} u_i \, v_i^H = UV^H$$

   Otherwise the block has to be calculated and stored as **dense** matrix block



low rank approximation of dense matrix for asymptotical smooth kernel function

Fast BEM - H-Matrix compression

Block approximation, frequency dependency

- Approximation of kernel function $\kappa(x,y)$ by truncated series expansion

$$\kappa(x,y) = \sum_{\nu=1}^{l} \varphi_\nu(x)\phi_\nu(y) + R_k(x,y)$$

  - approximation costs of matrix block depend on separation rank l
  - ideal: exponential decay of error term $R_k(x,y)$
  - exponential decay of error term is given for Laplace kernel $\kappa(x,y) = \frac{1}{4\pi r}$ if

$$R_k(x,y) = C_1\eta^l \;\rightarrow\; \frac{\min(\mathrm{diam}(X_t),\mathrm{diam}(X_s))}{\mathrm{dist}(X_t,X_s)} \leq \eta$$

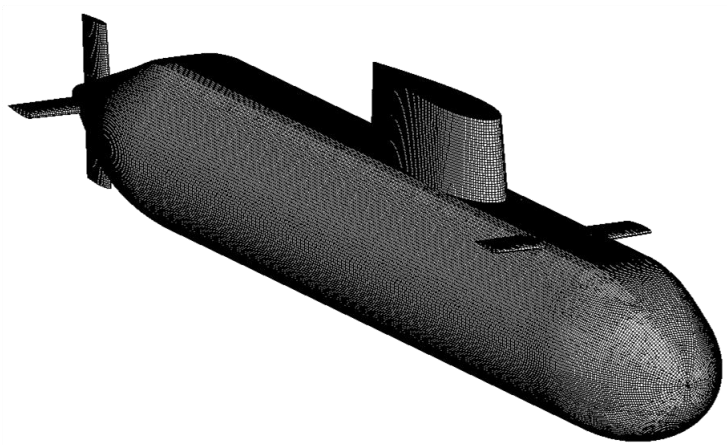
  - exponential decay of error term is given for Helmholtz kernel $\kappa(x,y) = \frac{1}{4\pi r}e^{-ikr}$ if

$$R_k(x,y) = C_2\eta_k^l \;\rightarrow\; (1 + \boldsymbol{k}\cdot\mathrm{dist}(X_t,X_s))\frac{\min(\mathrm{diam}(X_t),\mathrm{diam}(X_s))}{\mathrm{dist}(X_t,X_s)} \leq \eta_k$$

- Problems in the high-frequency range (k*diam(Γ) >> 1)
  - separation rank l depends linear on k ➔ block ranks as well ($l \sim k$)
  - admissable blocks getting smaller and smaller ➔ Matrix compression less efficient, memory costs increase

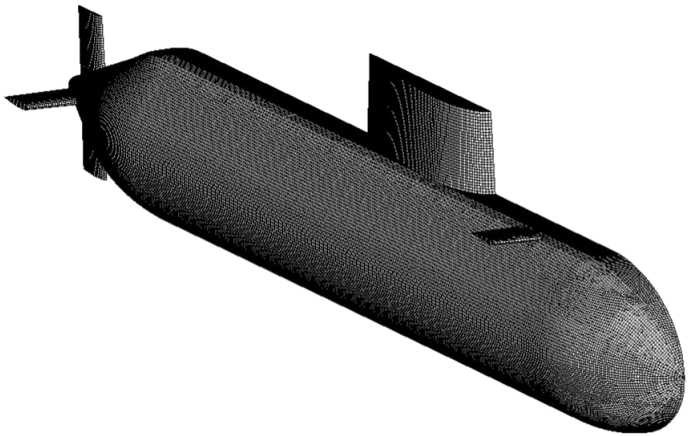
BeTSSi TES simulation
Model parameters

- **10 Elements/λ**



Frequency	n (DOFs)	k	ka
1kHz	115.738	4.2	252
3kHz	893.140	12.6	756
5kHz	1.980.562	20.9	1254

- **symmetry plane leads to block symmetric matrix structure:**



$$A = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}$$

⇒ Compression of 50% (time & memory)

BEM TES calculation (sound hard)

$$\left(\frac{1}{2}\mathrm{I} + \mathrm{K} - \boldsymbol{\alpha}\mathrm{D}\right)\mathrm{p} = \mathrm{p}_{\mathrm{i}}^{\mathrm{inc}} - \boldsymbol{\alpha}\mathrm{q}_{\mathrm{i}}^{\mathrm{inc}} \qquad \boldsymbol{\alpha} = {}^{\mathbf{i}}/_{\mathbf{k}} \text{ or } \mathbf{NtD}$$

➡ Each incident plane wave i leads to a RHS

- **Monostatic:**

$$\mathrm{TES} = 20\log_{10}\left(\frac{|\mathrm{r_j} - \mathrm{r_0}|\mathrm{p}_{\mathrm{scat}}^{\mathrm{j}}}{\mathrm{p}_{\mathrm{inc}}^{\mathrm{i}}}\right)$$

  - Multiple RHS $(\boldsymbol{j} = \boldsymbol{i})$
    - 0°-180° ⇒ 1800 ($\Delta\alpha_i = 0.1°$)
  - Single field point evaluation
  - **Reduction** of RHS by QR decomposition
    - 1kHz: 1800 => 125 ($\varepsilon = 10^{-5}$)
    - 3kHz: 1800 => 321 ($\varepsilon = 10^{-5}$)

- **Bistatic:**

$$\mathrm{TES} = 20\log_{10}\left(\frac{|\mathrm{r_j} - \mathrm{r_0}|\mathrm{p}_{\mathrm{scat}}^{\mathrm{j}}}{\mathrm{p}_{\mathrm{inc}}^{\mathrm{i}}}\right)$$

  - Single RHS $(\boldsymbol{j} \neq \boldsymbol{i})$
  - Multiple field point evaluation p_{scat}^{j}
    - 0°-360° ⇒ 3600 ($\Delta\alpha_j = 0.1°$)

Acceleration of GMRES solver for the Helmholtz BEM

MVP-based

- Deflated GMRES variants (restart)
- Damping of small eigenvalues (improvement of eigenvalue clustering) e.g. Subspace Recycling

analytic approach

- Regularization (BM, OSRC)
- Operator preconditioning

algebraic approach

- Preconditioning
- Approximation of the inverse system matrix (e.g. LU-Factorization of low precision: H-LU)

BeTSSi TES simulation
Iterative solution strategies

Novicos
Noise and Vibration Concepts

- **BM – no precond.**
  - System matrix

$$A = \left(\frac{1}{2}I + K - \frac{i}{k}D\right) = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}$$

    - 50% memory & time reduction

- **BM – HLU-precond.**
  - System matrix

$$A = \left(\frac{1}{2}I + K - \frac{i}{k}D\right) = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}$$

    - no memory & time reduction
  - Preconditioner, $\delta = 10^{-1}$

$$P = L_\delta U_\delta \approx A$$

- **BM – Block Jacobi HLU-precond.**
  - System matrix

$$A = \left(\frac{1}{2}I + K - \frac{i}{k}D\right) = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}$$

    - 50% memory & time reduction
  - Preconditioner, $\delta = 10^{-1}$

$$L_\delta U_\delta \approx A_1 \qquad P = \begin{pmatrix} L_\delta U_\delta & \\ & L_\delta U_\delta \end{pmatrix}$$

- **OSRC – Operator precond.**
  - System matrix

$$\tilde{A} = A + B$$

$$A = \left(\frac{1}{2}I + K\right) = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}$$

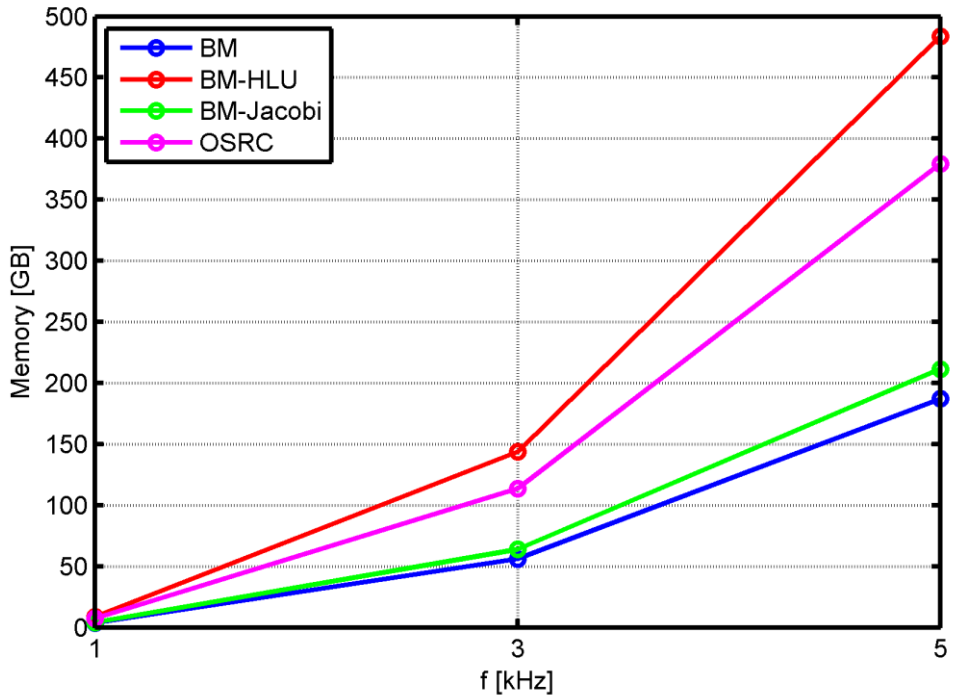
$$B = (NtDD) = \begin{pmatrix} B_1 & B_2 \\ B_2 & B_1 \end{pmatrix}$$

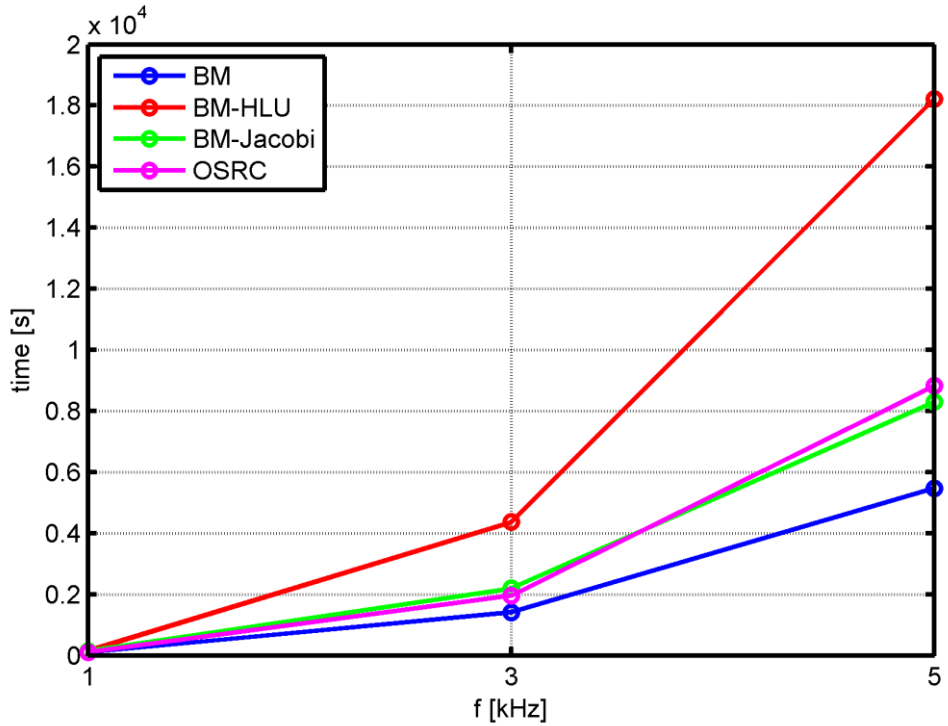
    - no memory & time reduction
  - Preconditioner

$$NtD$$

BeTSSi TES simulation – solution strategies

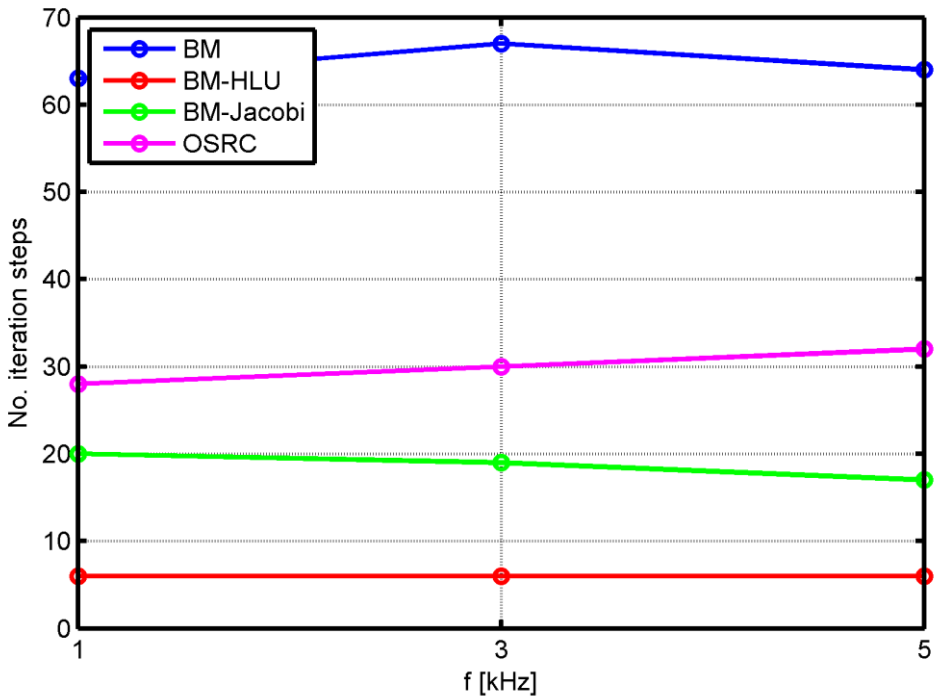
Memory requirements and setup times

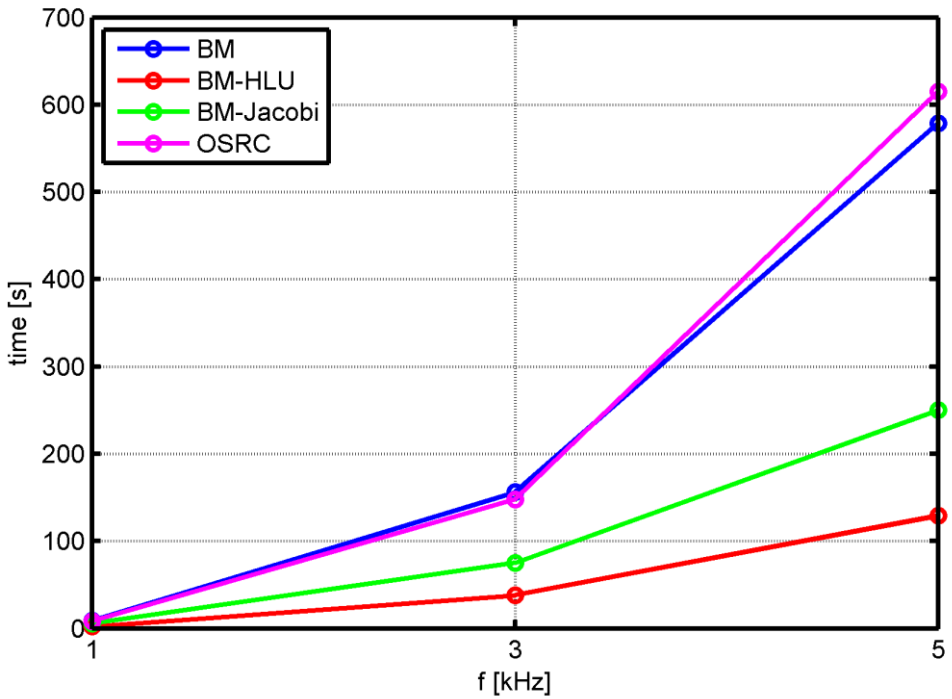




BeTSSi TES simulation – solution strategies

Iteration steps and solution time/rhs

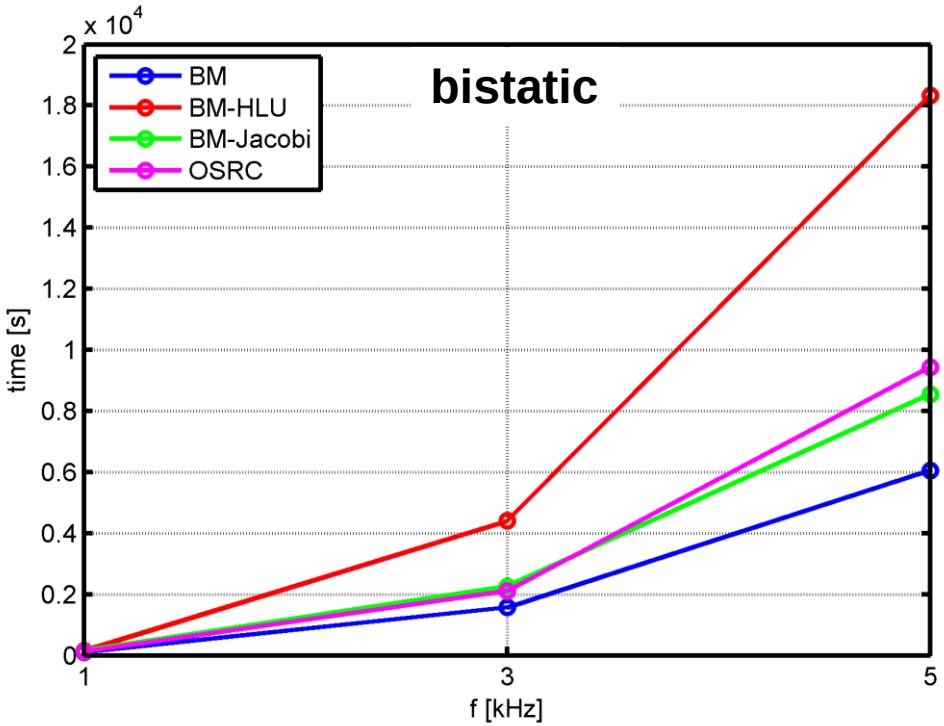


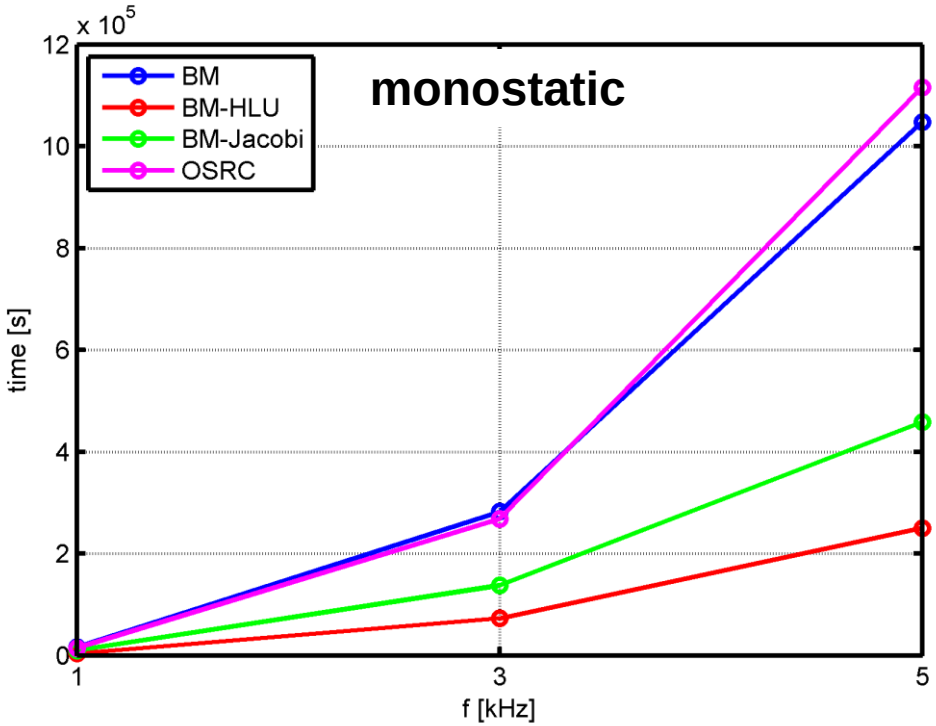


- Solution time/rhs reflects the number of iteration steps except for the OSRC regularisation.
- OSRC slows down due to doubled Matrix-Vector-Product (MVP) costs (separate matrix handling).

BeTSSi TES simulation – solution strategies

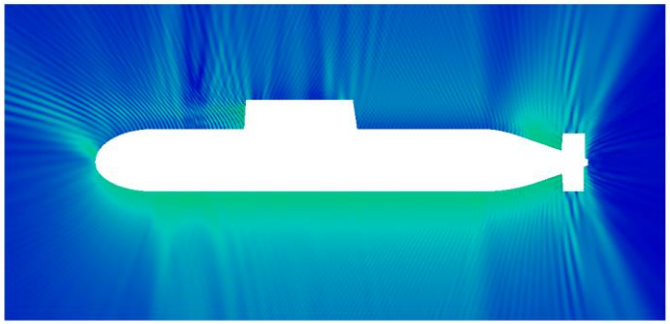
Overall computing time: bistatic vs. monostatic



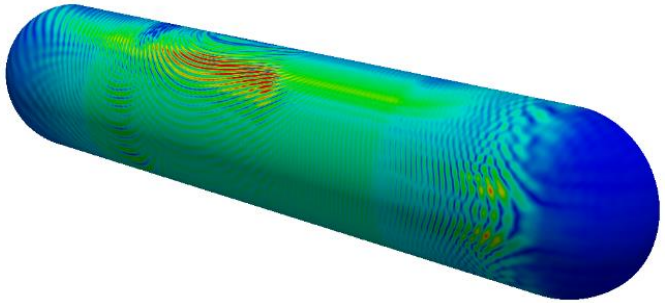


- Time for bistatic TES computation is determined by matrix assembly times.
- Overall time for monostatic TES computation is determined by solution time/rhs.

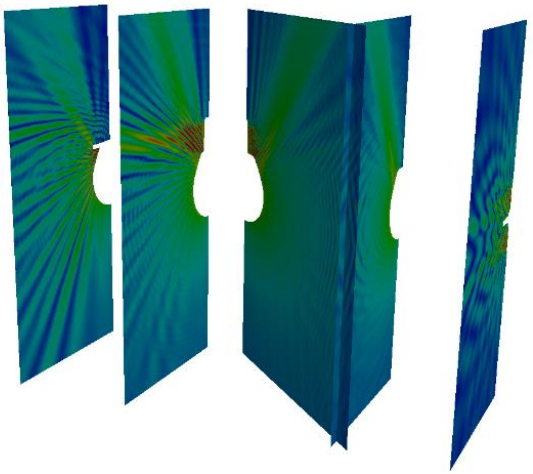
Field point evaluation
meshes



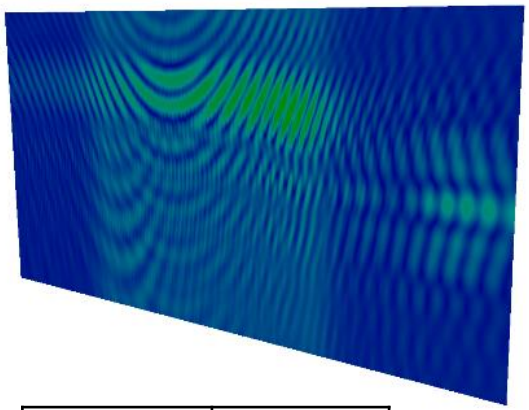
Frequency	DOFs
1kHz	198.051
3kHz	1.766.599



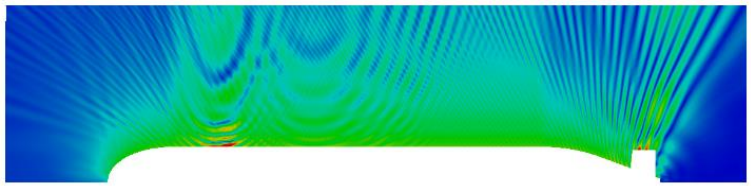
Frequency	DOFs
1kHz	114.767
3kHz	1.122.455



Frequency	DOFs
1kHz	188.143
3kHz	2.428.688



Frequency	DOFs
1kHz	150.348
3kHz	1.460.334



Frequency	DOFs
1kHz	86.397
3kHz	907.387

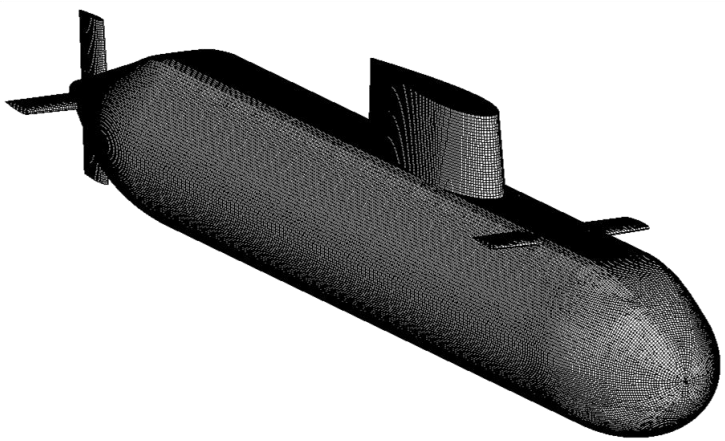
Total	
Frequency	DOFs
1kHz	737.706
3kHz	7.685.463

BeTSSi TES simulation
Summary

- H-Matrix approach extends the application range of the BEM, well suited for monostatic TES simulations (many rhs).
- Exploiting symmetric surfaces reduces setup costs for H-matrices in the same way as for the standard BEM.
- OSRC preconditioning effectively reduces the number of iteration steps for the TES simulation, but suffers from doubled MVP costs.
- Efficient field point evaluation by H-Matrix approach provides deeper insights into the radiation properties.
- BEM system matrix structure can be exploited for multiple objects of the same kind.

BeTSSi TES with Fast BEM techniques
Outlook

- Memory and time reduction
  - H^2-Matrix, DH^2-Matrix
    ➔ also applicable for field point evaluation
- Improved parallelisation ($(D)H^2$-Matrix)
  - OpenMP, MPI, GPU
- Solution costs
  - Butterfly algorithms



Frequency	n (DOFs)	k	ka	
1kHz	115.738	4.2	252	H-Mat
3kHz	893.140	12.6	756	H-Mat
5kHz	1.980.562	20.9	1254	H-Mat
10kHz	7.922.248	41.9	2514	H^2-Mat
20kHz	31.688.992	83.8	5028	DH^2-Mat
30kHz	71.658.074	125.7	7542	DH^2-Mat?

Novicos
Noise and Vibration Concepts

Thank you for your attention.